Math 1B
Post Midterm 3 Review and Final Exam Comments

Use the study guides from midterms 1, 2 and 3 to review chapters 5, 6, 7 (except 7.7) and 8.1, 8.2, 8.5.
The following questions act as a review for the section 7.7 and chapter 9 material which will be included.

[1] Write differential equations for the following situations. Make the signs of your constants clear in the equations.

[a] In a chemical reaction, chemical X is being broken down at a rate proportional to the square of the existing amount of the chemical.
[b] The water in a cylindrical tank is being drained out of a hole in its base. The height of the water in the tank is changing at a rate proportional to its square root.
[c] Your retirement account is growing at a rate proportional to the existing balance. In addition, you are continuously depositing additional funds into the account at a rate of \$5000 per year.
[d] A function is increasing at a rate inversely proportional to its square.
[e] The deceleration of an object due to fluid resistance is proportional to the object's velocity.
(Fluid resistance works in the direction opposite to which an object is moving, and causes the object to slow down.)

[2] Solve the following initial value problems.

[a] $\frac{dy}{dx} = \frac{2y}{x^3}, \quad y(1) = 1$

[b] $\frac{dy}{dx} = \frac{1+y^2}{\cos^2 x}, \quad y(0) = 1$

[c] $\frac{dy}{dx} = e^{2x+y}, \quad y(0) = 1$

[d] $\frac{dy}{dx} = \frac{1}{x^2 y}, \quad y(1) = 4$

[3] Use Euler's method to approximate the value of $y(2)$ for each initial value problem using the specified value of h.

[a] $\frac{dy}{dx} = x + y^2, \quad y(1) = 1, \quad h = 0.5$ WITHOUT USING A CALCULATOR

[b] $\frac{dy}{dx} = \cos x + \sin y, \quad y(0) = 0, \quad h = 0.2$

[c] $\frac{dy}{dx} = x^2 - 2y^2, \quad y(0) = 0, \quad h = 0.1$

[4] Estimate $\int_1^9 f(x)\,dx$ using $n = 4$ and each of the methods below.

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	7	9	10	13	12	10	7	3	2	2	5

[a] Midpoint Rule [b] Trapezoidal Rule [c] Simpson's Rule

[5] Consider the definite integral $\int_1^{25} \frac{1}{\sqrt{x}}\,dx$.

[a] Find L_8 and M_8. Find R_8, T_8 and S_{16} using the shortcuts discussed in lecture and the textbook.
[b] Using the error bounds formulae in the textbook, find the minimum number of subintervals needed to approximate the integral within 0.0001 using
[i] Midpoint Rule [ii] Trapezoidal Rule [iii] Simpson's Rule
[c] Using S_{40} (using the INTEGRAL program) and the error bounds formulae, find the best interval that the integral lies in.

The final exam will be approximately 50% multiple choice, with no partial credit for those problems (since you won't have to show work).
There will be a longer no-calculator section and a shorter calculator-allowed section.
The questions on volume will all be on the multiple choice calculator-allowed section.

You will be expected to simply set up the integrals, then use fnInt to find the correct answer.
That means **you must be able to set up the integrals correctly**, and **you must be able to use your calculator correctly**.

Answers

[1] [a] $X' = -kX^2$ [b] $h' = -k\sqrt{h}$ [c] $\frac{dA}{dt} = kA + 5000$ (t in years)

[d] $y' = \frac{k}{y^2}$ [e] $v' = -kv$

[2] [a] $y = e^{1-\frac{1}{x^2}}$ [b] $y = \tan\left(\frac{\pi}{4} + \tan x\right)$

[c] $y = 1 + \ln 2 - \ln(2 + e - e^{2x+1})$ [d] $y = \sqrt{18 - \frac{2}{x}}$

[3] [a] 4.75 [b] 2.3783 [c] 1.2565

[4] [a] 62 [b] 63 [c] $63\frac{1}{3}$

[5] [a] $L_8 \approx 9.4925$ $R_8 \approx 7.0925$ $M_8 \approx 7.8766$ $T_8 \approx 8.2925$ $S_{16} \approx 8.0152$

[b] [i] M_{2079} [ii] T_{2940} [iii] S_{234}

[c] $[7.8875, 8.1143]$